Turing Oscillators: Theory and Experiment

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Introduction

In 1952 Alan Turing published his seminal paper *The Chemical Basis of Morphogenesis* outlining his theory for the development of physical characteristics such as Tiger stripes and Leopard spots from a reaction diffusion system during the organisms development. This well known result is often referred to as Turing Patterns. What is less well known is that Turing's paper actually predicts six different results from a reaction system diffusion and that Turing Patterns are what he refers to as case "d" or "stationary waves of finite wave-length". Our works relates to studying another important conclusion, that of case "f" or the "oscillatory case with extreme short wave-length". This case predicts that a ring of oscillatory cells with nearest neighbor coupling will settle into a final state with neighboring cells π out of phase. Our experimental system is the first system able to directly test Turing's predictions (that we are aware of).

Experimental Overview

The experimental system we use is a microscale emulsion of Belousov-Zhabotinsky (BZ) oscillators. The BZ reaction is a complicated nonlinear chemical reaction with several formulations and models. The important characteristics for our work are that the reaction has a periodic oxidation spike which releases a diffusive inhibitory chemical upon spiking and that the reaction is photo inhibitable by inclusion of a specific light sensitive catalyst. Below is a figure demonstrating the behavior of two coupled BZ oscillators.

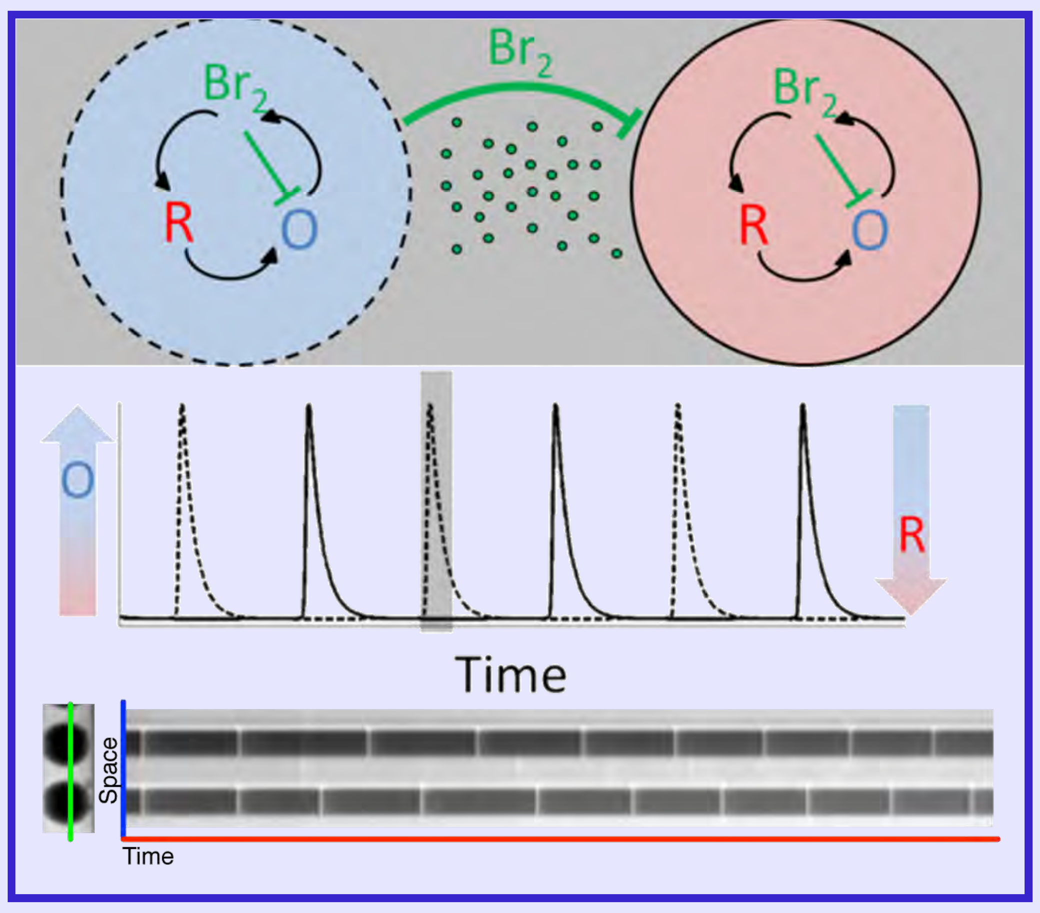

Figure 1: A schematic diagram demonstrating some of the key characteristics of the BZ reaction. The top portion gives a physical picture of two aqueous BZ reactions separated by an oil gap. When the cyclic reaction oxidizes (blue) it releases an inhibitory chemical that is able to diffuse across the oil gap and inhibit the oxidation of the neighboring drop. The middle portion demonstrates a time trace of the oxidation states of the two reactions. When the two reactions are coupled like this they automatically begin to oscillate out of phase with each other due to the inhibitory coupling. The bottom portion is a space-time plot of two BZ emulsion oscillators that were started initially in phase. During the course of the experiment the phases of the two oscillators drift until they settle out of phase.

Emulsion Overview

An emulsion of BZ oscillators is created by separating the aqueous drops with oil gaps. The emulsions are created using a flow-focusing microfluidic device that controls the flow of surfactant doped oil and the BZ reactants such that they form a monodispersed aqueous-in-oil emulsion. The emulsion is introduced to rectangular capillaries by capillary action with the ends sealed in epoxy.

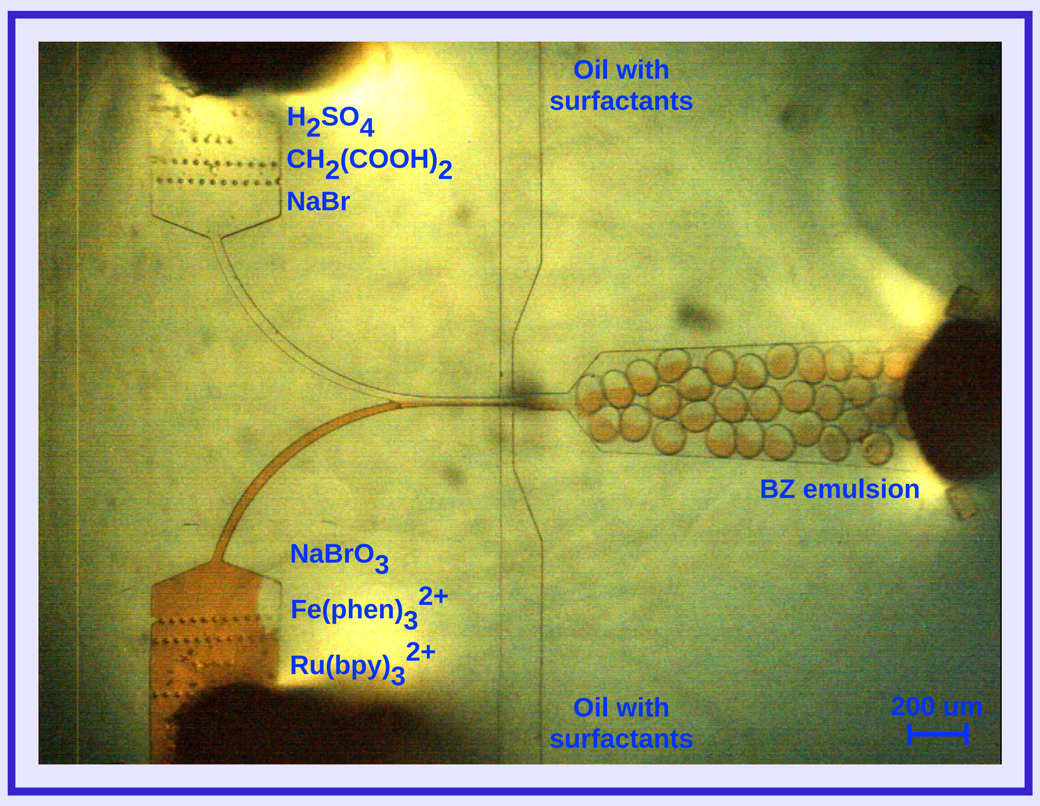

Figure 2: A flow-focusing microfluidic device used for creating the BZ emulsion. The BZ reactants are introduced from the left and pinched off into droplets by oil coming from the top and bottom. The emulsion then travels off the chip to the right for loading into capillaries.

Experimental Apparatus

The BZ emulsion is viewed and perturbed using a Programmable Illumination Microscope (PIM). The PIM consists of three arms with independent functions. The first arm illuminates the sample with Köhler Illumination and the second arm is a CCD microscope focused onto the sample area. The third arm is a modified commercial projector with modified projection optics that focus the image onto the microscope sample. The PIM can selectively illuminate individual BZ droplets within the emulsion and silence the oscillations within that droplet. A silenced droplet acts as a boundary of constant chemical conditions or inhibitor sink. Droplets that are separated by silenced drops are effectively isolated from one another.

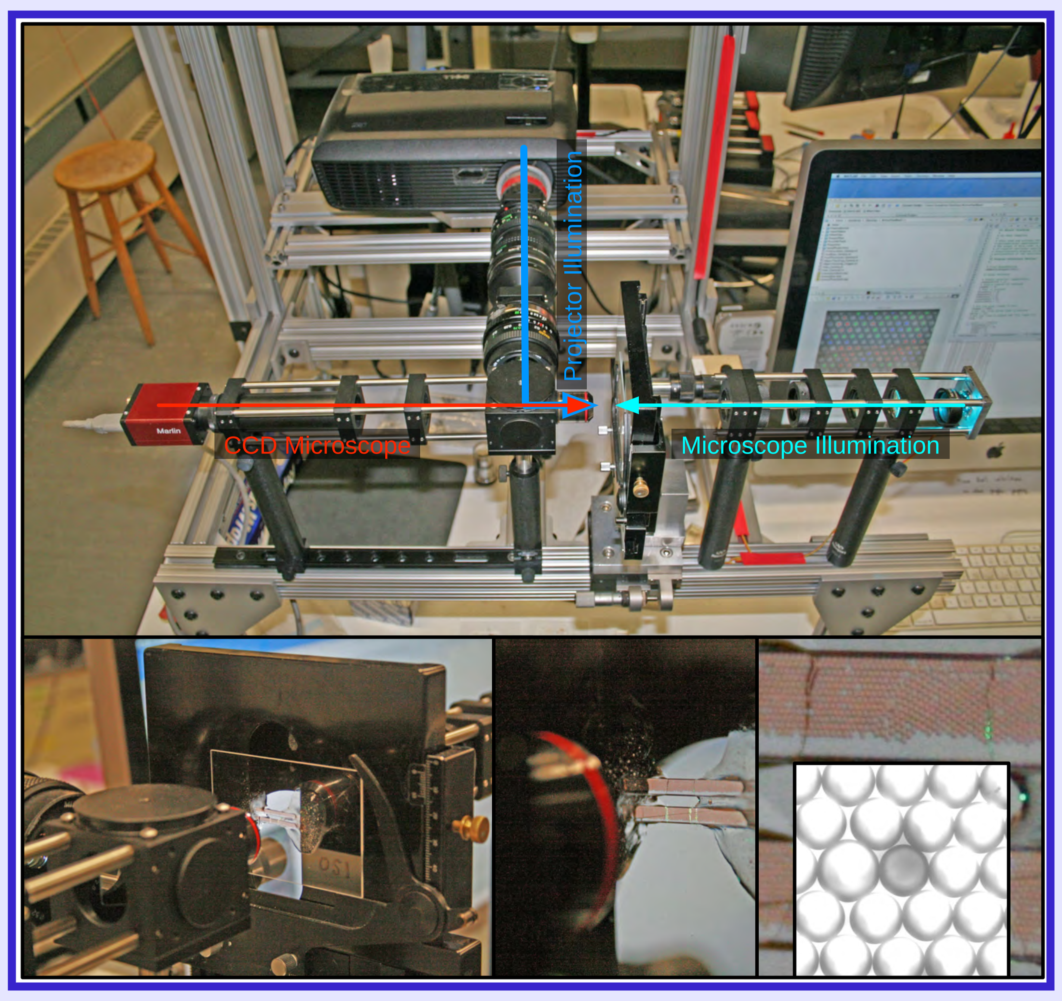

Figure 3: The Programmable Illumination Microscope with light paths highlighted and insets showing the sample area and capillary of emulsion being imaged. The bottom right inset is a micrograph from the PIM showing one drop that is not being illuminated/silenced in an array of others that are. The central droplet is freely oscillating in isolation.

Theory and Modeling

The BZ micro-oscillators were modeled using locally coupled Kuramoto oscillators placed on a lattice. The equations governing the phase of the oscillators are:

$$\dot{\phi}_i = \omega_i + K \sum_{\langle ij \rangle} \sin(\phi_i - \phi_j) \tag{1}$$

Here, the oscillator at site i is coupled locally to its nearest neighbors $\{j\}$. The intrinsic frequency of the individual oscillators is given by ω_i. This frequency is chosen randomly from a Gaussian distribution with zero mean and variance σ, and is a source of quenched disorder in this system. We study the specific case of the coupling strength, $K > 0$, in which every oscillator wants to be completely anti-aligned (π out of phase) with its nearest neighbor. We can then ask what role the geometry plays on the long time phase configuration of a lattice of these oscillators.

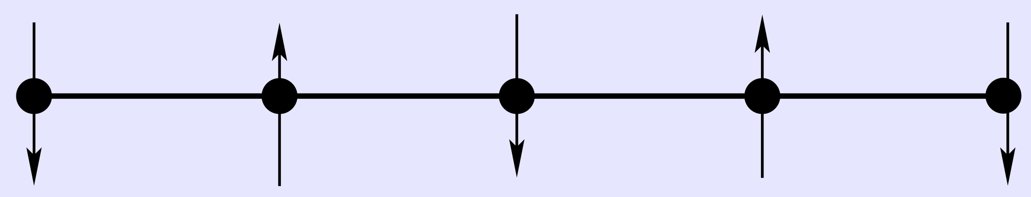

Figure 4: Schematic representation of anti-aligned phase oscillators described by the Kuramoto model.

To determine the long time phase patterns for a ring of oscillators, we perform a linear stability analysis on the set of ordinary differential equations given by equation 1. For simplicity we set all frequencies, ω_i, equal to one another. This corresponds to the limit of infinite coupling strength. With the constraint that the phase differences between neighboring oscillators around the ring must sum to 2π, we find the fixed point phase differences to be

$$\Delta\phi = \frac{2\pi m}{N} \tag{2}$$

where N is the number of oscillators in the ring, and m is any integer. When $K > 0$, only values of $\Delta\phi$ in the left half of the unit circle are attracting and the most strongly attracting phase difference is that which is closest to π. Thus, for even numbered rings the oscillators can always get into their desired anti-phase configuration, but for odd numbers they must compromise. For example, for a ring of three oscillators we predict a phase difference of $\Delta\phi = \pm 2\pi/3$, both of which are equally likely, and for five oscillators we predict $\Delta\phi = \pm 4\pi/5$.

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Turing Rings

Turing rings are created by photo-inhibiting all of the drops except for those in the desired rings. These inhibited droplets will act as a boundary of constant chemical conditions and otherwise not interact with the uninhibited drops. The freely oscillating drops are then equivalent to a one dimensional ring of droplets.

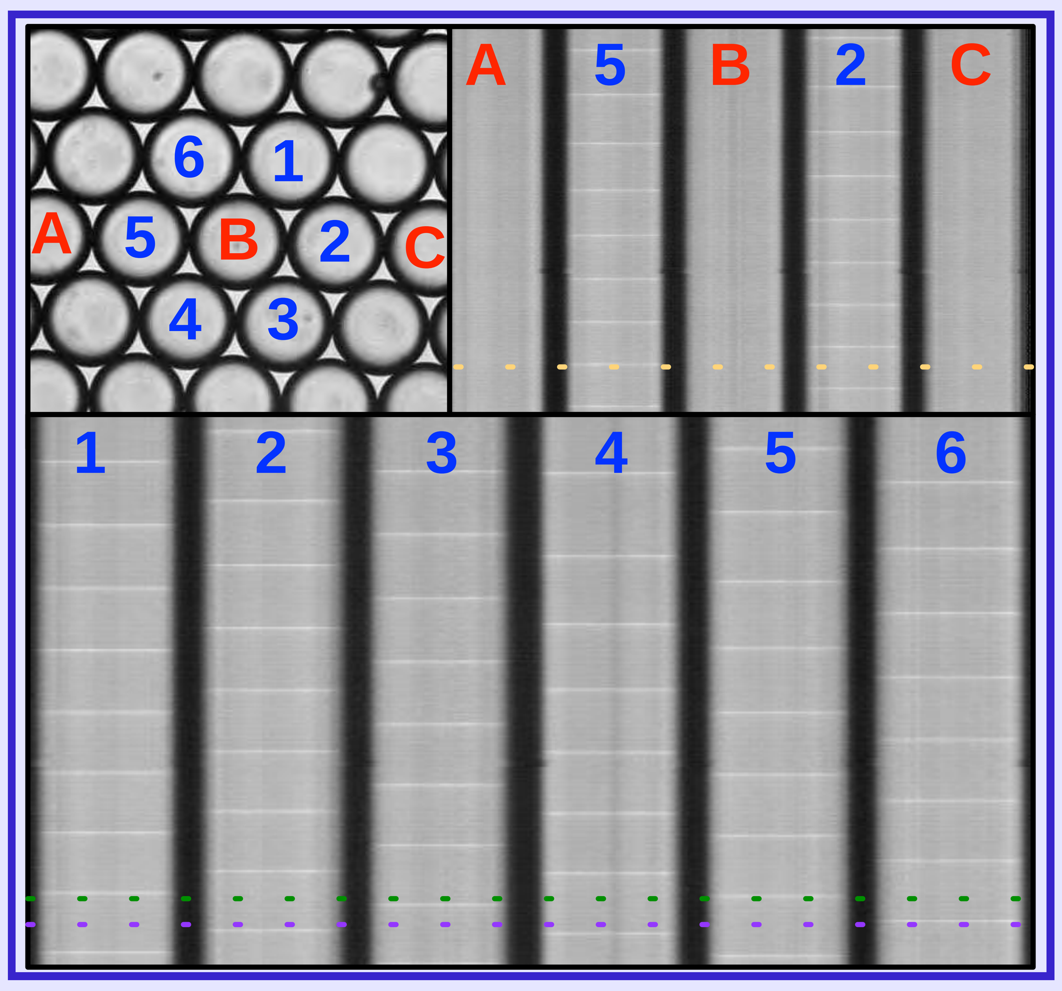

Figure 5: An emulsion experiment where a ring of droplets (labelled 1-6) are allowed to oscillate while all of the rest have been optically silenced. The top right image is a space-time plot taken horizontally across the center of the system demonstrated that the six droplets of the ring are oscillating while the others are silent. The bottom image is a space-time plot around the ring of oscillators. The green dashed line demonstrates that drops 1, 3, and 5 are in phase while the purple dashed line demonstrates that drops 2, 4, and 6 are in phase. The orange dashed line demonstrates that the two phase clusters including drops 2 and 5 are mutually out of phase.

Turing's paper predicts that a ring of N oscillators should preferentially form two phases clusters of $N/2$ members each that are mutually out of phase. However for rings where N is odd this is not a viable solution. For these frustrated cases the system instead assumes a gramoidal pattern with one droplet spiking at a time following a $(N-1)\pi/N$ nearest neighbor phase difference. Observationally these patterns resemble triangles, pentagrams, etc.

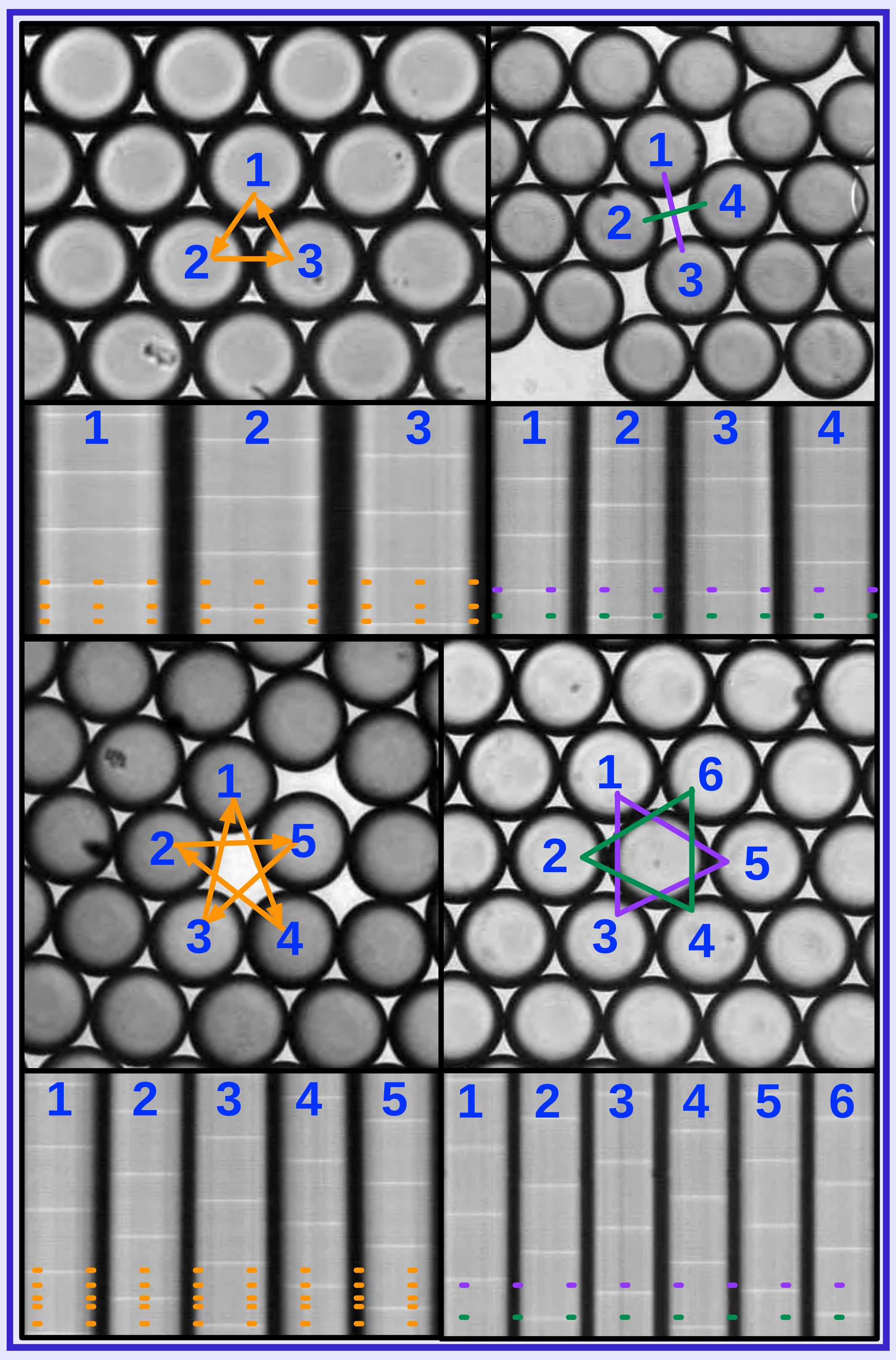

Figure 6: Multiple emulsion experiments with Turing rings of 3, 4, 5, or 6 oscillators. In each case the upper image shows the droplets that have been allowed to oscillate while the lower image is a space-time plot of the oscillating drops. The two odd numbered cases on the left both demonstrate the gramoidal pattern with the dashed orange lines indicating the sequence of oscillations. The two even numbered cases on the right both demonstrate two anti-phase phase clusters with the dashed green and purple lines indicating the two clusters.

References

M. Giver, Z. Jabeen, B. Chakraborty *Phys. Rev. E, 83 (2011)*

J. Delgado et al. *Soft Matter, 7 (2011)*